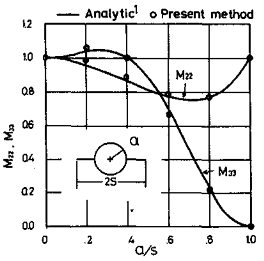

Fig. 2 Apparent-mass coefficient of planar midwing with body.

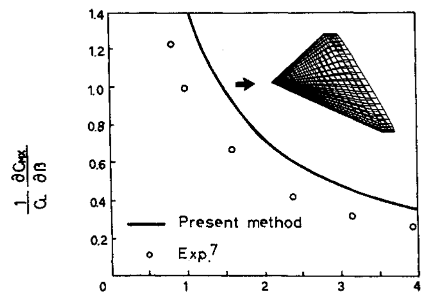

Fig. 3 Rolling moment due to sideslip of delta wings vs aspect ratio Λ.

Table 1 Stability derivatives of flat triangular wing[a]

	Analytic[1]	Present method
C_{yp}	0.209	0.211
$C_{z\alpha}$	−3.49	−3.51
C_{zq}	−2.09	−2.11
$C_{l\beta}$	−0.105	−0.104
C_{lp}	−0.218	−0.214
C_{lr}	−0.0157	−0.0156
C_{mq}	−0.471	−0.474
C_{np}	−0.0157	−0.0158

[a]Aspect ratio: 2.22; angle of attack: 0.1 rad.

Finally, the rolling moment due to the sideslip of the delta wings against the aspect ratio was computed to compare calculated results with experimental data.[7] As shown in Fig. 3, the profile of the cross section was an ellipse with a thickness ratio of 0.05 and was divided into 48 segments. In Fig. 3, the solid line denotes the calculated data and the "circle" means the experimental results (Ref. 7, p. 187). It is shown that the rolling moment due to the sideslip decreases greatly with the decreasing aspect ratio, and that agreement between measurements and calculation is good.

In summary, a simple computational method for the slender body theory has been developed to provide a computer-aided design tool for aerospace engineers who want to estimate the stability derivatives of an airplane with a slender body configuration at its initial design stage. It is finally noted that since the stability derivatives related to the drag force cannot be calculated by the preceding potential-flow theory, we use a semiempirical formula[1] for estimating the drag.[8]

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Compressed Polynomial Approach for Onboard Ephemeris Representation

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I. Introduction

ONBOARD ephemeris representation, be it for spacecraft pointing,[1] online annotation of remotely sensed imagery,[2] or NAVSTAR/global positioning system navigation message generation for a user satellite,[3] is becoming an increasingly important demand for any mission. In all of these cases, the end user is not only concerned with the accuracy of the ephemeris information but also the frequency and load of ground updates and onboard processing time. The alternatives for onboard ephemeris generation/representation known to date are 1) modeling of all the perturbative forces onboard,[4] 2) conic propagation using the delta-rho perturbation method,[5] 3) modeling of Keplerian parameters plus perturbations,[6] and 4) approximating the ephemeris by polynomials and transmitting the coefficients.[3]

As discussed in Ref. 4, although the first option allows more autonomy for the spacecraft, limited computing power onboard and ground update transmission limitations might render the approach unattractive sometimes. The second alternative, viz the delta-rho perturbation method,[5] is a very elegant approach, whose applicability to geosynchronous orbit has been demonstrated with simulation studies. In this method the trajectory of a desired orbit is treated onboard as a dispersion around a reference orbit, whose trajectory is generated in a large, ground-based computer and is transmitted to the spacecraft. Since this involves integration of differential equations, it may be beyond the capacity of the limited processing power of onboard processors. The third method, as stated in Ref. 6, does not abruptly deteriorate in accuracy after the duration of applicability but involves lengthy algorithms requiring large storage requirements. The suitability of the fourth option, namely polynomial approximation, has been demonstrated by Wakker et al.[3] for dissemination of NAVSAT ephemerides between both the mission center and the regional center as well as the regional center and user spacecraft. There the representation was restricted to parts of an orbit. All the same, the polynomial approach per se has been discarded by many[6] as being either incapable of representing multiple orbits or as too demanding in terms of transmission load when extended to multiple orbits. No evidence of exercising this computationally simple approach, with a view to compress the representation further, is seen in the literature, at least to the authors'

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knowledge. It is to fill this lacuna that the avenue was extensively explored, which resulted in the approach proposed in this Note based on the compression of the coefficients of the approximating polynomials.

II. Approach

The fundamental assumption made by this approach, termed the "compressed coefficients method," is the availability of proven ground software that provides the ephemeris, which starts from a set of definitive orbital elements and takes into consideration all of the relevant perturbative forces. Also, the ephemeris generated is assumed to be accurate enough to the end user until the next orbit determination is carried out (say, e.g., two days for a sun-synchronous type of mission). Let L be the number of orbits up to which the ground ephemeris meets the user's accuracy requirements. Now, the problem at hand is to represent this ephemeris as closely as possible onboard, keeping the frequency and load of transmission message to a minimum and using limited onboard memory and processing time.

Let T be the period of the orbit calculated using the definitive orbital elements and $[\boldsymbol{R}_1(t)\colon 0 \le t \le T]$ be the trajectory of the first orbit in the series of L orbits to be represented onboard. Following Schneider and Trexel,[5] the first orbit is taken as the "reference orbit" and the subsequent orbits as the "dispersed orbits," whose deviations from the reference orbit would now be approximated by polynomials as described below.

Let $[\boldsymbol{R}_i(t)\colon 0 \le t \le T]$ be the trajectories of the dispersed orbits for $i = 2,\ldots, L$. At any time t, the deviations of each of the dispersed orbits from the reference orbit vary systematically from one orbit to the next. If we denote $\boldsymbol{\rho}_i(t)$ as the "deviation vector" given by

$$\boldsymbol{\rho}_i(t) = \boldsymbol{R}_i(t) - \boldsymbol{R}_1(t); \qquad 0 \le t \le T \tag{1}$$

then $\boldsymbol{\rho}_i(t_k)$ is a systematic function of i for any t_k in the interval $0 \le t \le T$. As shown in Fig. 1, which gives the plots of the x coordinate of the position vector and the deviation for a few revolutions of a sun-synchronous satellite, it can be seen that the latter can be represented by a lower-degree polynomial than the former, and further, the variation of the deviation vectors from one revolution to the next follows a monotonic trend. Therefore, if each of these deviation vectors are represented by polynomials, the coefficients of the polynomials would vary systematically from one orbit to another and hence can themselves be further compressed by polynomial coefficients. Putting this mathematically,

$$\boldsymbol{\rho}_i(t) = \sum_{n=0}^{M} C_n(i) T_n(x) \text{ for } i = 2,\ldots, L \tag{2a}$$

where T_n are Chebyshev polynomials and

$$x = (2t - T)/T; \qquad 0 \le t \le T \tag{2b}$$

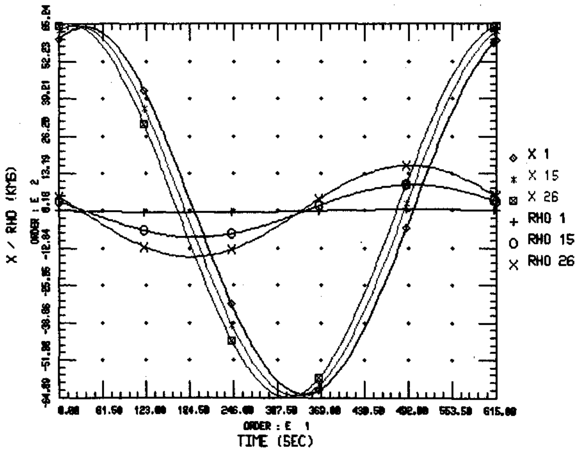

Fig. 1 Plots of X and rho; numbers indicate orbit numbers.

The coefficients $C_n(i)$ are obtained, as suggested in Ref. 7, by minimizing the maximum error in absolute value over a finite set of reference points, which are more crowded at the end points so that the Gibbs phenomenon of high amplitude rapid oscillations at the ends can be avoided. Coefficients $C_k(i)$, $K = 0,1\ldots, M$ vary systematically with i and can themselves be approximated using polynomials in i by

$$C_k(i) = \sum_{l=0}^{P} Q_{kl} T_l(x) \qquad \text{for} \qquad k = 0, \quad 1\ldots M \tag{3a}$$

where T_l are again Chebyshev polynomials and

$$x = (2i - 2 - L)/(L - 2) \tag{3b}$$

with i as the revolution number.

Further, the reference trajectory can be represented to the required accuracy by

$$\boldsymbol{R}_1(t) = \sum_{m=0}^{N} C_{1m} T_m(x) \tag{4a}$$

with

$$x = (2t - T)/T; \qquad 0 \le t \le T \tag{4b}$$

The entire ephemeris covering L revolutions is now compressed to

$$\{c_{1m}\}_{m=0}^{N}, \; \{[Q_{kl}]_{k=0}^{M}\}_{l=0}^{P}$$

a set $N + MP$ values for each position component. As noted in Ref. 3, it is not necessary to fit and transmit velocity components separately as they can be obtained directly by differentiating the corresponding position components without any degradation in accuracy. The velocity vector component can be calculated using the deviation vector

$$\boldsymbol{\rho}_i'(t) = \sum_{n=0}^{M} C_n(i) T_n'(x) \tag{5a}$$

where

$$T_0' = 0$$

$$T_1' = 2/T$$

$$T_n'(x) = 2x T_{n-1}'(x) + 4/T \; T_{n-1}(x) - T_{n-2}'(x) \tag{5b}$$

and the reference velocity vector

$$\boldsymbol{R}_1'(t) = \sum_{m=0}^{N} C_{1m} T_m'(x) \tag{6a}$$

where

$$T_0 = 0$$

$$T_1 = 2/T$$

$$T_m'(x) = 2x T_{m-1}'(x) + 4/T \; T_{m-1}(x) - T_{m-2}'(x) \tag{6b}$$

Once the C_{1m} and Q_{kl} coefficients are available, the state vector at any time t_A can be reached through the following steps.

1) Find the quotient q and remainder t_r of t_A with T, the orbital period (for better accuracy T can be repeatedly calculated for each of the orbits as suggested in Ref. 5) using $q = \text{Int}(t_A/T)$ and $t_r = t_A - q \times T$.

2) Fetch the compressed coefficients Q_{kl} from memory and substitute $i = q$ in Eq. (3b) and calculate coefficients $C_k(i)$, $k = 0,1,\ldots, M$ using Eq. (3a).

3) Substitute $t = t_r$ in Eq. (2b) and calculate $\rho_q(t_r)$ and $\rho_q'(t_r)$ by substituting for $C_n(i)$ in Eqs. (2) and (5), respectively.

4) Fetch C_{1m} and calculate $\boldsymbol{R}_1(t_r)$ and $\boldsymbol{R}_1'(t_r)$ by substituting $t = t_r$ in Eqs. (4b) and (6b), respectively.

5) The sum of $\rho_q(t_r)$ and $\boldsymbol{R}_1(t_r)$ and the sum of $\rho_q'(t_r)$ and $\boldsymbol{R}_1'(t_r)$ give the position and velocity vectors, respectively, at t_A.

III. Case Study

A case study was carried out for a 900-km, sun-synchronous orbit with 14 orbits per day by generating the ephemeris for a period of 2 days using a force model that takes into account forces due to asphericity of the Earth, atmospheric drag, lunisolar gravitational attraction, and solar radiation pressure. It was found that with 12 coefficients per component for the reference orbit and the deviation vector, with 4 coefficients representing the variation of these 12 coefficients of the deviation vector over 28 orbits, the final accuracy was about 300 m. It is to be noted here that by accuracy is meant the deviation of the component calculated using the fitted coefficients from the value given in the original ephemeris. This accuracy can be improved by increasing the number of coefficients. When the number of coefficients was increased to 14, both for the reference orbit as well as the deviation vector, the final accuracy achieved was about 30 m. As for the number of words for transmission, in the case of 12 coefficients, the number of words (32 bits) needed is 180, and in the case of 14 coefficients, it would be 210. With a typical transmission rate of 50 bits/s,[6] the total transmission time would be around 2 min. On the other hand, if a single polynomial is to be fitted for the duration of 28 periods, assuming that it is possible to do so, the total number of words needed for a similar accuracy would be around 750 if the results presented in Table V of Ref. 7 are extended. Regarding the time required for computing a single-state vector onboard with the proposed approach, it would be about 15–20 ms, which makes it adoptable for real-time attitude determination for spacecraft pointing also.

IV. Conclusions

A procedure for ephemeris representation of multiple orbits for onboard applications is presented. The approach is based on the representation of the ephemeris of any given orbit in terms of the approximating polynomial of a reference orbit and a compressed approximating polynomial of the deviation of the given orbit from the reference orbit. This is shown to be economical in terms of ground update load and frequency as well as onboard computation load and memory. The efficacy of the method is demonstrated with application to a sun-synchronous type of orbit. It is envisaged that this procedure might prove to be an effective tool in reducing the frequency of ground updates in the case of geosynchronous orbits yielding accuracy that is limited only by the ground ephemeris prediction software.

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